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Perceptual Independence of Size and Weight by Dynamic Touch

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Historically, the existence of a size–weight illusion has led to the conclusion that perceptions of size and weight are not independent. A dependence of perceived heaviness on physical volume (perceptual separability), however, is different from a dependence on perceived volume (perceptual independence). Three experiments were conducted to evaluate perceptual independence. The relations between perceived size and weight and physical size and mass were evaluated in Experiment 1. Perceived weight, length, and width were structured only by the corresponding physical variables, whereas variations in volume were not separable from variations in mass. F. G. Ashby and J. T. Townsend's (1986) test for perceptual independence was applied in Experiment 2. Perceived weight was independent of perceived length and volume. Experiment 3 used a magnitude estimation paradigm to investigate the extent to which information–perception relations could be related to the observed patterns of separability and independence.

In the course of many common activities (such as using eating utensils, various tools, or athletic equipment), one is able to perceive various properties of objects through grasping and lifting. Such perceptions are often achieved by means of the muscle's sensory capabilities—a sensibility known classically as the muscle-sense and more recently as kinesthesia. From the earliest investigations of the muscle-sense, two kinds of perceptions have been identified. The first was the perception of weight (Weber, 1834/1978). Weber demonstrated that observers were better at discriminating among the masses of objects when they were actively lifted rather than passively rested in the hand. This demonstration indicated that observers used the sensory capabilities of their muscles in generating a perception of weight. A second perception attributable to the muscle-sense was the perception of an object's length. Hoisington (1920) noted that individuals were remarkably good at judging the length of an object that they could hold but not see. Because observers only held these objects and did not run their hands along the lengths of the stimuli, this perception of size is also achieved using the muscle's sensory capabilities.

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Historically, these two perceptions have been thought to be dependent on one another, in large part because of the influence of both mass and volume on perceived weight. This effect is commonly known as the size–weight illusion. The size–weight illusion is a phenomenon in which, across a set of stimuli with equal masses, an increase in physical volume results in a decrease in perceived heaviness (Charpentier, 1891; see Ellis & Lederman, 1993, for a purely haptic size–weight illusion). The response correlation (lack of statistical independence between responses) associated with the size–weight illusion has led to the development of models within which perceived weight is a function of perceived size. Accordingly, haptic perceptions of size and weight would not be independent. Two prominent examples of such models are the information-integration model (Anderson, 1970, 1972) and the expectation model (Ross & Gregory, 1970). In both models, perceived heaviness is a function of certain cognitive processes that combine initial impressions of size and weight. The differences between these particular models, as well as those variations that have followed, are how and why these two perceptions are dependent on one another. Despite their differences, however, this class of models, based on a presumed lack of perceptual independence between size and weight, has figured prominently in the study of weight perception (see Jones, 1986).

Making an inference about perceptual dependence on the basis of an observed response correlation is not, however, a straightforward process. In developing a formal test of perceptual independence, Ashby and Townsend (1986) pointed out that making a perceptual response is a multifaceted process. Perceptual (in)dependence represents only one of a number of influences on the resulting perceptual report. Observers also use some decision rules to choose which of the available responses corresponds to their perception of the object (see, e.g., signal detection theory; Green & Swets, 1966). Two properties may be perceived independently, but a particular decision rule may result in a lack of independence in the participant's reports; this effect is termed *decisional separability*.[1] Likewise, an observed response correlation could be the result of a failure of perceptual separability. Perceptual separability refers to the effect of variations in a physical dimension, such as volume, on a noncorresponding perception, such as the perception of weight. Two properties are said to be perceptually separable if the perception of one remains unchanged across variations in the other. These distinctions indicate that the observed lack of response independence associated with the size–weight illusion may not necessarily be the result of a lack of perceptual independence.

Although related to perceptual and decisional separability, perceptual independence only refers to the influence of one perception on another. Two perceptions are independent when the "perception of each [property] is in no way contingent upon or interacts with the perception of the other" (Ashby & Townsend, 1986, p. 154). That is, not only will the mean values of each perception remain unchanged across levels of the other (zero correlation), but the entire distribution will also remain unchanged. Formally, then, independence is defined according to the following equation:

$$f(x_i, y_j)_k = f(x_i)_k f(y_j)_k, \tag{1}$$

where x and y are two perceptions at particular levels i and j, respectively. Here, independence between perceptual variables exists when, for a given stimulus k, the joint distribution of two responses equals the product of the distributions of each response alone (see Ashby & Townsend, 1986).

A test of perceptual independence that distinguishes between the effects of one percept on another and perceptual separability requires a paradigm in which both perceptions are reported on every presentation of each stimulus (Ashby & Townsend, 1986; Garner & Morton, 1969). With very few exceptions (Ellis & Lederman, 1993), however, there has been a lack of reported research in which haptic perceptions of both weight and size were reported simultaneously. The present experiments were designed to evaluate the independence of perceived size and weight. In Experiment 1, the relations between perceived size and weight and the physical sizes and masses of the stimuli were evaluated. Psychophysical techniques involving the comparison of conditional probabilities and nonmetric correlations among variables were used in Experiment 2 to formally evaluate perceptual independence (Ashby & Townsend, 1986; Garner, 1962; Garner & Morton, 1969; McGill, 1954). A similar analysis from Ashby and Maddox (1991) has previously been applied to demonstrate a lack of independence between haptic perceptions of shape and texture (Reed, 1994). In Experiment 3, a magnitude estimation paradigm was used to investigate the extent to which information–perception relations could be related to and possibly support the observed patterns of separability and independence from Experiments 1 and 2. To limit discussion to the haptic perceptual system (Gibson, 1966), participants were not allowed to view the stimuli. The haptic perceptual system relies on the activity of the set of mechanoreceptors in the muscles, skin, tendons, and joints; Loomis and Lederman (1986) termed this *tactual perception.*

In the present experiments, participants used a particular subsystem of the haptic perceptual system known as dynamic touch. Dynamic touch relies primarily on the use of the muscle as a sensory organ (Gibson, 1966; Turvey & Carello, 1995b). This class of touch is very similar to what Loomis and Lederman (1986) termed *kinesthetic perception,* but the term *dynamic touch* emphasizes the identification and analysis of phenomena at the functional level of the event dynamics—that is, the dynamics of grasping, lifting, holding, and moving (Turvey & Carello, 1995a). Analysis at this level generally focuses on the patterns of forces, motions, and inertial properties associated with lifting and holding. Dynamic touch is distinguished from the other two subsystems of the haptic perceptual system: the cutaneous and haptic subsystems (Gibson, 1966). Cutaneous touch is the kind of touch where perceptions are achieved solely by means of skin deformations. Gibson (1966) defined the haptic subsystem of the haptic perceptual system as relying on the combined use of skin and joint receptors, as might occur when the hand is run along a surface. However, the relatively weak contributions of joint receptors to perception along with the demonstration that joint postures can be perceived through dynamic touch (Pagano & Turvey, 1995) suggest that this subsystem may rely more on the combined use of skin and muscle receptors. This is similar, in fact, to the definition of haptic touch that was offered by Loomis and Lederman (1986) as being the combined use of cutaneous and kinesthetic inputs. The potential influence of the cutaneous or haptic subsystems on the present experiments was restricted by having participants hold all stimuli by a handle.

Experiment 1

The relations between perceived size and weight and the physical sizes and masses of the stimuli were explored in Experiment 1. A set of stimuli were used in which the dimensions of mass, volume, length, and width all varied. The stimuli were chosen in order that half were greater than, and half were less than, the standard on each dimension. Participants were allowed to hold, but not to see, these objects. On each trial, the standard and one stimulus were presented, and the observer was asked to report which of the two was greater on one of the four dimensions. An uncertainty analysis (e.g., Garner, 1962) was used to evaluate the patterns of contingency. This analysis measured the amount of structure in the observed responses that was provided by one or more of the predictors (see the Appendix). The expectation was that each response would be strongly dependent on the corresponding physical property but that a

[1] Throughout the article, it is necessary to distinguish between a perception of a particular property and the report corresponding to that perception. Terms already exist for the study of weight perception and are used as follows: Mass is the physical property of the stimulus, perceived weight is the perception of that property, and perceived heaviness is the report corresponding to that perception.

lack of separability could result in a codependence on another property.

Method

Participants. Fifteen undergraduate students (12 men, 3 women) at the University of Connecticut participated in this experiment as a means of fulfilling a course requirement. Of these participants, 13 were right-handed and 2 were left-handed. All participants reported no problems with the normal use of their hands or limbs.

Design. Participants reported perceived differences in weight, volume, length, and width for pairs of hand-held stimuli that they could wield but not see. The eight stimuli and one standard were a subset of the full set of objects described in Table 1. The reports in this experiment were forced choices of which stimulus was greater on the given dimension. The participants were not told that one of the stimuli in the pair was always the standard. The eight experimental stimuli were designed so as to create a 2 (mass) × 2 (volume) × 2 (style of volume change) factorial design. The two levels of mass and volume indicate either greater than or less than the standard on that dimension. The style-of-volume-change variable categorizes the volume changes as being due to variations either in length or width. This design allows for the effects of variations in size (volume, length, or width) to be considered independently of the effects of mass variations. Additionally, the effects of variations in volume attributable to variations in length can be separated from those attributable to variations in width.

Apparatus. A set of eight stimuli and an additional standard were created in which mass, volume, length, and width varied independently. These objects were styrofoam cylinders with handles made from wooden dowels of 1.24-cm diameter (see Figure 1). The wooden dowels ran through the length of the cylinder and extended

Table 1
Physical Dimensions of the Stimuli in Experiments 1–3

Number	Mass (g)	Volume (cc)	Length (cm)	Width (cm)	$\log I_1$ ($g \cdot cm^2$)	$\log I_3$ ($g \cdot cm^2$)
1[a]	309	1,520	12.0	12.6	5.34	3.77
2	309	3,792	12.0	20.0	5.35	4.16
3[a]	309	15,096	12.0	40.0	5.39	4.74
4[a]	309	1,520	4.8	20.0	5.23	4.15
5	309	3,792	12.0	20.0	5.35	4.16
6[a]	309	15,096	48.0	20.0	5.80	4.13
7	460	1,520	12.0	12.6	5.52	3.95
8	460	3,792	12.0	20.0	5.53	4.34
9	460	15,096	12.0	40.0	5.57	4.93
10	460	1,520	4.8	20.0	5.40	4.33
11	460	3,792	12.0	20.0	5.53	4.34
12	460	15,096	48.0	20.0	5.98	4.34
13[a]	660	1,520	12.0	12.6	5.68	4.11
14	660	3,792	12.0	20.0	5.69	4.50
15[a]	660	15,096	12.0	40.0	5.73	5.09
16[a]	660	1,520	4.8	20.0	5.56	4.49
17	660	3,792	12.0	20.0	5.69	4.50
18[a]	660	15,096	48.0	20.0	6.15	4.51
Standard						
Exp. 1, 2	460	3,792	12.0	20.0	5.53	4.34
Exp. 3	368	2,033	10.0	16.0	5.39	4.04

Note. Two standards were used, one for Experiments 1 and 2 and a second for Experiment 3. Measures of length were for the cylinder only (all cylinders had a 12-cm handle), whereas all other measures were for the entire stimulus. Exp. = experiment.
[a]Stimuli for Experiments 1 and 2.

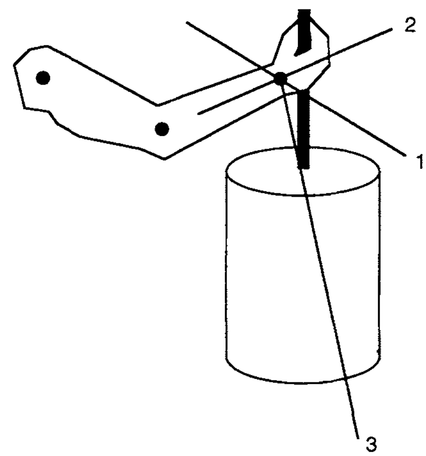

Figure 1. Orientation of the stimulus relative to the hand and arm. The three indicated axes are the symmetry axes (eigenvectors) of the stimulus rotating about the wrist.

12 cm out from one end to serve as a handle. The cylinders were composed of styrofoam disks (2.4-cm thick) that were glued together. The masses of the stimuli were manipulated independently of the size dimensions (length, width, and volume) with the addition of lead shot throughout the layers of styrofoam disks. This was accomplished by hollowing out portions of the disks and distributing the lead shot and cotton evenly throughout the entire volume of the cylinder. Special care was taken to ensure that the additional mass was distributed from the center to the outer edges of the cylinder so that the cylinder would have a nearly uniform density. The volume of the objects increased as a function of either an increase in the width or the length of the styrofoam cylinder. The particular values of length and width were chosen to make the volumes of the stimuli equal across the two styles of volume change. The dimensions of all eight stimuli and the standard are provided in Table 1. Because all of the handles were of equal size, the measures of size in Table 1 refer to the cylinder alone, whereas the measures of mass and I_{ij} refer to the entire stimulus.

The participant sat at a student desk that was raised up on a platform approximately 35 cm above the floor. The purpose of the platform was to reduce any perceived range restriction on the lengths of the stimuli, which were held with the styrofoam cylinder extending below the right hand (see Figure 1). A wooden board clamped to the desk extended out to the participant's right-hand side and provided support for the participant's right forearm. The right hand was placed through a slit in a floor-to-ceiling curtain hanging to the right of the participant's chair. This curtain occluded the hand and the stimulus without making the participant uncomfortable and without interfering with the free wielding of the stimulus.

Procedure. Upon entering the experimental room, the participant was seated at the desk behind the curtain. All of the stimuli were hidden from view for the entire session, and the participant knew nothing about the nature of the objects except that they had wooden handles. Each trial consisted of successively presenting a pair of stimuli to the participant and asking which of the pair was greater on a given dimension. The four perceived dimensions were weight, volume, length, and width. The instructions were to report "which of the pair is heavier," "which of the pair is larger," "which of the pair is longer," or "which of the pair is wider." For perceived volume trials, participants were instructed to make their report based "not only on length or width but on the overall volume or size of the object." The trials were conducted in four sessions, one for each dimension. All four sessions were run in one day, with a short break between sessions. The trials were spaced about 3–5 s apart, but participants were allowed to elect a short break between trials to avoid fatigue. Participants held each object only once on each trial, and they were instructed to make their best guess if they were unsure of the comparison. Participants wielded these occluded objects through motions solely about the wrist (a restriction that was imposed by the armrest). These wielding motions consisted primarily of twisting the stimulus along its longitudinal axis and swinging the stimulus from side to side or from front to back. The order of the sessions and of the trials within each session were randomized. Within a session, each stimulus was compared with the standard twice. On one of those trials, the standard was presented to the participant first, and on the other it was presented second. No time limits were made on the trials. All of the procedures in the present series of experiments conformed to the ethical guidelines of the American Psychological Association.

Results and Discussion

As described in the Appendix, the uncertainty analysis measures the amount of structure in observed responses that is provided by one or more predictor variables. Although many forms of partitioning are possible, a form parallel to analysis of variance (ANOVA) was chosen, in which the strength of each main effect and interaction was identified. The uncertainty terms represented the strength of (or amount of uncertainty reduction due to) the relation between variables inside the parentheses. The main effects have a colon designating the effect of the variable(s) to the right side of the colon on the variable(s) to the left side of the colon. Interaction terms are designated with a bar across all relevant variables. Independent variables are in capital letters, and response variables are in lowercase letters.

Perceived heaviness. The number of times that response m_i (where the subscript refers either to greater than or less than the standard) was made in each of the eight conditions is tabulated in Table 2, along with the results on the other perceptual dimensions. The effects of the physical stimulus variables, mass (M), volume (V), length (L), and width (W) are presented in this table. The manipulations of V are shown separately as those resulting from a variation in L and those resulting from a variation in W. Thus, V is represented by collapsing across the style-of-volume-change variable (L or W). Recall from Table 1 that L_s and W_t (where the subscript refers either to greater than or less than the standard) were so configured in order that the volume for L_- would be equal to the volume of W_- and that the volume of L_+ would be equal to the volume of W_+. Thus, V_- could be substituted for L_- and W_- in the matrix and V_+ could be substituted for L_+ and W_+ so that the data matrix would represent the effects of M and V. The analysis proceeds by evaluating separately the overall effects of volume, MV, and the effects of variations in either length, ML, or width, MW.

The results of the uncertainty analyses are shown in Table 3 (the degrees of freedom for each effect are listed in the Appendix). The values of the uncertainty terms represent the amount of relation between perceived heaviness, m, and the physical variables of mass, M, and size, B (where B is either V, L, or W). In each of the three analyses, there was a significant relation between m and the set of predictor variables: $U(m{:}\ M, V)$, $-2\log\lambda = 263.43$, $p < .005$; $U(m{:}\ M, L)$, $-2\log\lambda = 132.00$, $p < .005$; $U(m{:}\ M, W)$, $-2\log\lambda = 140.15$, $p < .005$. The partitioning of the $U(m{:}\ M, B)$ terms shows that the main effect of M, $U(m{:}\ M)$ was significant in each analysis: $B = V$, $-2\log\lambda = 257.91$, $p < .005$; $B = L$, $-2\log\lambda = 124.87$, $p < .005$; $B = W$, $-2\log\lambda = 136.38$, $p < .005$. Size, however, contributed little, if anything, to the patterning of m. None of the main effects of size, $U(m{:}\ B)$, was significant ($p > .05$). Finally, although the significance of the interactions between M and B cannot be evaluated, these uncertainty terms were relatively small in each of the three analyses. Especially when compared with the amount of structure provided by mass alone, it does not appear that perceived heaviness was a function of size, either directly or through an interaction with mass.

Perceived volume. The number of reports of v_h (where the subscript refers either to greater than or less than the standard) in each of the eight conditions is reported in Table 2. The uncertainty analyses on the v data are shown in Table 3 (the degrees of freedom for each effect are listed in the Appendix). By looking at the effects of the stimulus variables on perceived volume, $U(v{:}\ M, B)$, one can see that the values were significant: $U(v{:}\ M, V)$, $-2\log\lambda = 95.50$, $p < .005$; $U(v{:}\ M, L)$, $-2\log\lambda = 57.03$, $p < .005$; $U(v{:}\ M, W)$, $-2\log\lambda = 42.37$, $p < .005$. A distinction between the results of m and v is seen in the partitioning of $U(v{:}\ M, B)$. For the m analysis, the observed structure in the responses was provided by the appropriate stimulus variable, M. For v, variations in size contributed to perceived volume: $U(v{:}\ V)$, $-2\log\lambda = 29.75$, $p < .005$; $U(v{:}\ L)$, $-2\log\lambda = 24.64$, $p < .005$; $U(v{:}\ W)$, $-2\log\lambda = 8.22$, $p < .005$, as did variations in M ($B = V$, $-2\log\lambda = 56.21$, $p < .005$; $B = L$, $-2\log\lambda = 24.64$, $p < .005$; $B = W$, $-2\log\lambda = 31.22$, $p < .005$). These results indicate that both physical mass and physical volume contribute to perceived volume.

Table 2
Frequency of Reports on the Four Perceptual Dimensions of Heaviness (m), Volume (v), Length (l), and Width (w) in Experiment 1

Report	M_-L_-	M_-L_+	M_+L_-	M_+L_+
m_-	30	24	0	0
m_+	0	6	30	30
v_-	27	9	9	1
v_+	3	21	21	29
l_-	30	4	22	2
l_+	0	26	8	28
	M_-W_-	M_-W_+	M_+W_-	M_+W_+
m_-	29	30	0	2
m_+	1	0	30	28
v_-	27	17	10	4
v_+	3	13	20	26
w_-	25	8	16	5
w_+	5	22	14	25

Note. Subscripts refer to greater than (+) or less than (−) the standard. The stimuli varied in mass (M) and volume (V). Volume was manipulated through variations either in length (L) or width (W).

Table 3
Uncertainty Analysis for the Effects of Mass (M) and Size (B) on Reported Perceptions of Heaviness (m), Volume (v), Width (w), and Length (l) in Experiment 1

Source	Uncertainty		
	$B = V$	$B = L$	$B = W$
$U(m{:}\ M)$	0.78*	0.76*	0.83*
$+\ U(m{:}\ B)$	0.00	0.01	0.00
$+\ U(mMB)$[a]	0.02	0.04	0.03
$=\ U(m{:}\ M, B)$	0.80*	0.81*	0.86*
$U(v{:}\ M)$	0.17*	0.15*	0.19*
$+\ U(v{:}\ B)$	0.09*	0.15*	0.05*
$+\ U(vMB)$[a]	0.03	0.05	0.02
$=\ U(v{:}\ M, B)$	0.29*	0.35*	0.26*
$U(w{:}\ M)$			0.03
$+\ U(w{:}\ W)$			0.17*
$+\ U(wMW)$[a]			0.01
$=\ U(w{:}\ M, W)$			0.21*
$U(l{:}\ M)$		0.02	
$+\ U(l{:}\ L)$		0.48*	
$+\ U(lML)$[a]		0.06	
$=\ U(l{:}\ M, L)$		0.56*	

[a]Probability not calculated.
$^*p < .05$.

Perceived width. The number of reports of w_j (where the subscript refers either to greater than or less than the standard) as a function of variations in M and W is presented in Table 2. Although data were collected on the other four conditions, they are not reported here because there is not yet a reason to expect any important effects of L on w when W is unchanging across variations in L. The results of the uncertainty analysis are shown in Table 3 (the degrees of freedom for each effect are listed in the Appendix). As before, there was a significant effect of the stimulus variables on w: $U(w{:}\ M, W)$, $-2\log\lambda = 34.22$, $p < .005$. The partitioning of $U(w{:}\ M, W)$ showed that the only significant main effect was the relation between w and W: $-2\log\lambda = 27.93$, $p < .005$. There was no significant main effect of M ($p > .05$), and the interaction uncertainty was relatively small. Perceptions of width do not appear to be a function of variations in mass.

The distinction between perceived volume and width as a function of variations in width is potentially important. The only difference between $U(w{:}\ M, W)$ and $U(v{:}\ M, W)$ was the perceptual report (the stimuli were identical). For these stimuli, variations in W were equivalent to variations in V, and so the expected perceptual reports were identical as well. Therefore, the fact that participants could separate the effects of W and M when reporting w and not when reporting v indicates that perhaps there is a perception of the entire volume of the object that is distinct from the perception of the object's width (even when V and W completely covary, $B = W$).

Perceived length. The number of reports of l_i (where the subscript refers either to greater than or less than the standard) as a function of variations in M and L is presented in Table 2. The results of the uncertainty analysis on these data are presented in Table 3 (the degrees of freedom for each effect are listed in the Appendix). Once again, $U(l{:}\ M, L)$ was significant: $-2\log\lambda = 91.26$, $p < .005$. The partitioning of $U(l{:}\ M, L)$ shows that virtually all of the structure in l was being contributed by the relation between l and L: $-2\log\lambda = 78.86$, $p < .005$. The contingent uncertainty relating l to M was not significant ($p > .05$), indicating that reports of perceived length were not a function of variations in mass. As in the w analysis, there were differences between the results for l and v for $B = L$. For an identical set of stimuli, reporting l (instead of v) resulted in a relatively larger effect of the set of stimulus variables and a lack of significant effect of M.

Conclusions. Experiment 1 began to explore the question of how and when reports of perceived size and heaviness should be independent by identifying the relations between perceived size and heaviness and the actual sizes and masses of the stimuli. Reports of heaviness, length, and width were only contingent upon the appropriate stimulus variable. It appears, then, that variations of size and mass were separable in these cases. Perceived volume, however, was shown to be strongly contingent upon variations in mass. Within the general context of response independence, the present results are most directly related to the issue of perceptual separability. Whether the perceptions are independent was tested in Experiment 2, in which perceptions of both size and weight were reported on every trial.

The results of the present experiment raise an issue, not directly related to perceptual independence, regarding the relative strengths of each perception. In particular, there appears to be more structure to reports of perceived heaviness and length than to reports of perceived volume or width. Such results could possibly indicate that perceptions of weight and length are stronger, more primary, or more common than the others. At this point, however, definitive conclusions would be premature. Because of the constraints imposed by the materials and the need to make volume variations equivalent across variations in length and width, the ratios of change across each dimension were not equivalent. Although the present analysis was designed to be independent of the underlying metric (Garner & McGill, 1956), the potential effects of unequal ratios of change were untested. Additional research would be required in order to make definitive conclusions regarding the relative strengths of these perceptions.

Experiment 2

Evaluating perceptual independence in a way that distinguishes it from separability requires that both perceptions be reported on each trial (Ashby & Townsend, 1986; Garner & Morton, 1969). In this manner, the effects of variations in one perception on the level of another perception can be determined for a given stimulus. A complete identification experiment was used in Experiment 2, in which two potentially independent perceptions were reported for a set of stimuli varying orthogonally on the corresponding physical dimensions.

Two analyses were used to evaluate perceptual independence. First was the uncertainty analysis of Garner and Morton (1969). This analysis was similar to that used in Experiment 1 except that here there were two perceptual reports, m and b, whereas only one, y, was used before (see the Appendix for details). Garner and Morton's analysis has been shown to be more closely related to perceptual separability (Ashby & Townsend, 1986); therefore, Ashby and Townsend's analysis was used in order to make definitive conclusions regarding the perceptual independence of size and weight.

Ashby and Townsend's (1986) analysis was based on the definition of perceptual independence that was formally identified in Equation 1. For their purposes and those of the present analysis, the distribution of perceptual effects was represented by the conditional response probabilities. Therefore, Equation 1 translates to the criterion that the probability of producing a joint response at particular levels of two response variables will equal the probability of reporting that level of the first times the probability of reporting that level of the second. This test of response independence is termed *sampling independence.*

Ashby and Townsend's (1986) analysis provided the statistical and logical techniques for evaluating perceptual independence and distinguishing that from the other potential influences on a perceptual report. They demonstrated that an independence of two perceptual reports is indicative of perceptual independence only when the decision criteria remain constant across the levels of each perception (see their Theorem 1 and accompanying proof). The complete Ashby and Townsend analysis, then, measures response independence along with two tests of decisional separability. The first two tests, the tests of the partial contingent uncertainties and of marginal response invariance, relate to the decision criteria. The test of partial contingent uncertainties uses the uncertainty values from Garner and Morton's (1969) analysis to measure the relation between each response variable and the inappropriate stimulus variable after the effects of the appropriate stimulus variable have been partialed out. The second test, marginal response invariance (the separability test), is related to the first because it tests whether the probability of correctly reporting the level of one variable depends on the level of the other. This is tested by evaluating the following identities (Ashby & Townsend, 1986):

$$P(m_ib_1|M_iB_1) + P(m_ib_2|M_iB_1) = P(m_ib_1|M_iB_2) + P(m_ib_2|M_iB_2), \quad (2)$$

$$P(m_1b_j|M_1B_j) + P(m_2b_j|M_1B_j) = P(m_1b_j|M_2B_j) + P(m_2b_j|M_2B_j), \quad (3)$$

where b and B refer to perceived and actual size, respectively. If decisional separability holds along with perceptual separability, then the criteria of each test is met. Failure of decisional or perceptual separability could result in failure to meet the criteria of one or both tests, although it may not necessarily force such a failure. The third test, the test of sampling independence, requires decisional separability to hold for it to be logically related to perceptual independence. It does not require perceptual separability to hold.

Using the Ashby and Townsend (1986) analysis, one can conclude that two perceptions are independent only when the criteria of all three tests are met. If one or both of the first two tests fail, then perceptual independence may still hold but there is no test for it. If the first two tests hold but the third fails, then perceptual independence is rejected. Together, these tests offer a powerful indication of perceptual independence.

Method

Participants. Nine undergraduate students (8 men and 1 woman) at the University of Connecticut participated in this experiment as a means of fulfilling a course requirement. Of these participants, 8 were right-handed and 1 was left-handed. All participants reported no problems with the normal use of their hands or limbs.

Design and apparatus. The design and apparatus of the present experiment were identical to those of Experiment 1 with the following exceptions. Participants reported which of a pair of hand-held stimuli (experimental stimulus and standard) was greater on a pair of dimensions. One dimension was always heaviness, and the second was volume, length, or width. The dependent measure was the number of times that each of the four possible responses were made to each stimulus. The four responses (where b = perceived size) were m_-b_-, m_-b_+, m_+b_-, m_+b_+ (where the subscript refers to greater than [+] or less than [−] the standard on that dimension). The stimuli were identical to those used in Experiment 1 (see Table 1).

Procedure. The procedure was identical to that of Experiment 1 with the following exceptions. The experiment was conducted in three sessions in order to obtain reports on each of the three dimensions of size: volume, length, and width. Four comparisons were made for each stimulus in each session.

Results

Independence of perceived weight and volume. The number of times that response m_iv_h (where the subscripts refer to greater than or less than the standard on that dimension) was made in each of the eight conditions is shown in Table 4. The manipulations of V are shown separately as those resulting from a variation in L and those resulting from a variation in W. Thus, V is represented by collapsing across the style-of-volume-change variable (L or W). The analysis proceeds first by conducting Garner and Morton's (1969) analysis, followed by Ashby and Townsend's (1986) analysis.

The results of Garner and Morton's (1969) uncertainty analysis are shown in Table 5 (the degrees of freedom for each effect are listed in the Appendix). The uncertainty terms are of a similar form to those used in Experiment 1 except for the addition of subscripted terms indicating that the effects of those subscripted variables have been partialed out. The analysis was conducted separately for the stimuli with complete variations in volume, $B = V$, and for those in which the variation in volume was achieved purely through

Table 4
Frequency of Joint Reports of Perceived Heaviness (m) and Perceived Size (Where the Report Was Either Volume [v], Length [l], or Width [w]) in Experiment 2

Report	M_-L_-	M_-L_+	M_+L_-	M_+L_+
m_-v_-	27	6	0	0
m_-v_+	7	28	0	2
m_+v_-	2	1	25	12
m_+v_+	0	1	11	22
m_-l_-	31	3	1	0
m_-l_+	4	30	0	1
m_+l_-	1	0	33	1
m_+l_+	0	3	2	34
	M_-W_-	M_-W_+	M_+W_-	M_+W_+
m_-v_-	25	8	1	0
m_-v_+	11	25	0	5
m_+v_-	0	2	19	4
m_+v_+	0	1	16	27
m_-w_-	21	4	0	1
m_-w_+	14	29	0	6
m_+w_-	1	0	22	2
m_+w_+	0	3	14	27

Note. Subscripts refer to greater than (+) or less than (−) the standard. The stimuli varied in mass (M) and volume (V). Volume was manipulated through variations either in length (L) or width (W).

variations in length, $B = L$, or through variations in width, $B = W$. It is important to note that the size response in each analysis was always perceived volume.

The significant values of $U(m, v{:}\ M, B)$ in the bottom line of Table 5 indicate that there was an effect of the set of stimuli on the set of responses: $B = V$, $-2\log\lambda = 358.15$, $p < .005$; $B = L$, $-2\log\lambda = 189.11$, $p < .005$; $B = W$, $-2\log\lambda = 172.46$, $p < .005$. The partitioning of $U(m, v{:}\ M, B)$ shows that both of the direct contingencies between each response and the appropriate stimulus variable, $U(m{:}\ M)$ and $U(v{:}\ B)$, were significant in each of the three analyses. For $U(m{:}\ M)$: $B = V$, $-2\log\lambda = 280.00$, $p < .005$; $B = L$, $-2\log\lambda = 148.79, p < .005$; $B = W$, $-2\log\lambda = 131.76, p < .005$. For $U(v{:}\ B)$: $B = V$, $-2\log\lambda = 63.33, p < .005$; $B = L$, $-2\log\lambda = 35.16$, $p < .005$; $B = W$, $-2\log\lambda = 28.41, p < .005$. There was, however, no significant relation between the two responses: $U(m{:}\ v)$, $B = V$, $-2\log\lambda = 0.02, p > .10$; $B = L$, $-2\log\lambda = 0.68$, $p > .10$; $B = W$, $-2\log\lambda = 1.22$, $p > .10$. Relatedly, there was no significant relation between responses after the effects of the stimulus variables were partialed out: $U_{MB}(m{:}\ v)$, $B = V$, $-2\log\lambda = 8.9, p > .05$; $B = L$, $-2\log\lambda = 3.4$, $p > .10$; $B = W$, $-2\log\lambda = 4.41$, $p > .10$.

Evaluating the partial contingent uncertainties constitutes the first of the three tests in Ashby and Townsend's (1986) analysis of perceptual independence. The criterion that all values must not be significantly greater than zero was met with one exception. For $U_B(v{:}\ M)$: $B = V$, $-2\log\lambda = 2.63$, $p > .10$, $B = L$, $-2\log\lambda = 2.91, p > .10$; $B = W$, $-2\log\lambda = 4.62$, $p > .05$. For $U_M(m{:}\ B)$: $B = V$, $-2\log\lambda = 5.67, p > .05$; $B = L$, $-2\log\lambda = 2.74$, $p > .05$; $B = W$, $-2\log\lambda = 7.22, p < .05$. The one exception was $U_M(m{:}\ B)$ for $B = W$. However, this criterion was met for the data from the complete set of stimuli, $B = V$, meaning that, in the general case, participants' reports of heaviness were not significantly affected by variations in volume. This also held in the case in which the volume variations resulted from length manipulations.

The second test in Ashby and Townsend's (1986) analysis is the separability test conducted by applying Equations 2 and 3. Comparisons were among the probabilities of making a particular report at each level of the inappropriate stimulus variable. Each pair of conditional probabilities was compared by means of a binomial test corrected for continuity (Snedecor & Cochran, 1989). The results of this test for all of the data in Experiment 2 are summarized in Table 6. All of the tests for perceived heaviness and perceived volume ($b = v$) were nonsignificant ($p > .05$), indicating that participants could report variations in mass separately from

Table 5
Uncertainty Analysis for the Effects of Mass (M) and Size (B) on Perceived Heaviness (m) and Perceived Size (b) in Experiment 2

	Uncertainty				
	$b = v$			$b = l$	$b = w$
Source	$B = V$	$B = L$	$B = W$	$B = L$	$B = W$
$U(m{:}\ M)$	0.70*	0.75*	0.67*	0.75*	0.61*
$+\ U(b{:}\ B)$	0.16*	0.18*	0.14*	0.64*	0.23*
$+\ U_B(b{:}\ M)$	0.01	0.02	0.02	0.01	0.00
$+\ U_M(m{:}\ B)$	0.02	0.01	0.04*	0.01	0.06*
$+\ U_{MB}(m{:}\ b)$	0.02	0.02	0.02	0.00	0.01
$=\ U(m{:}\ b{:}\ M{:}\ B)$	0.91*	0.98*	0.89*	1.41*	0.91*
$-\ U(m{:}\ b)$	0.00	0.00	0.01	0.00	0.00
$=\ U(m, b{:}\ M, B)$	0.91*	0.98*	0.88*	1.41*	0.91*

Note. B and b refer to dimensions of volume, length, or width.
*$p \leq .05$.

Table 6
Results of the Separability Test on Perceived Heaviness (m) and Perceived Size (b) in Experiment 2

	$b = v$			$b = l$	$b = w$
Probability	$B = V$	$B = L$	$B = W$	$B = L$	$B = W$
$P(m_-\|M_-B_-)$	0.97	0.94	1.00	0.97	0.97
$P(m_-\|M_-B_+)$	0.93	0.94	0.92	0.92	0.92
z	0.78	0.00	1.18	0.51	0.51
$P(m_+\|M_+B_-)$	0.99	1.00	0.97	0.97	1.00
$P(m_+\|M_+B_+)$	0.90	0.94	0.86	0.97	0.81
z	1.82	0.72	1.28	0.00	2.39*
$P(b_-\|M_-B_-)$	0.75	0.81	0.69	0.89	0.61
$P(b_-\|M_+B_-)$	0.63	0.69	0.56	0.94	0.61
z	1.44	0.82	0.97	−0.43	0.00
$P(b_+\|M_-B_+)$	0.76	0.81	0.72	0.92	0.89
$P(b_+\|M_+B_+)$	0.78	0.67	0.89	0.97	0.92
z	0.01	1.07	−1.49	−0.51	−0.01

*$p \leq .05$.

variations in volume. This, along with meeting the criterion from the test of partial contingent uncertainties, indicates that the decisional separability held for the $B = V$ and $B = L$ stimuli. In contrast to the results of the previous test, however, the participants' behavior was unchanged across conditions for the $B = W$ stimuli.

The results of the test of sampling independence for all of the data in Experiment 2 are shown in Table 7. This test was conducted by applying the criterion of nonmetric statistical independence in Equation 1 to the conditional probabilities for each of the possible joint responses. Each pair of conditional probabilities was compared by means of a binomial test corrected for continuity (Snedecor & Cochran, 1989). A nonsignificant difference between the pair of conditional probabilities indicates that the two responses were independent for that stimulus. If the perceptions were independent, then the responses should have been independent in each comparison. The criterion of sampling independence was met for every comparison in each of the three perceived volume analyses ($p > .85$).

The evidence, then, favors a conclusion of perceptual independence of weight and volume in the general case of complete volume variations. The conclusion of independence also applied to the limiting case in which the stimuli varied only in length. With regard to the other limiting case in which stimuli varied only in width, the test of sampling independence showed only that responses of perceived heaviness and volume were independent. The presence of a significant partial contingent uncertainty, however, indicated that this test may not be logically related to perceptual independence.

Independence of perceived weight and length. The number of times that response $m_i l_j$ (where the subscripts refer either to greater than or less than the standard on that dimension) was made in each of the four conditions is tabulated in Table 4. The results of Garner and Morton's (1969) uncertainty analysis are shown in Table 5 (the degrees of freedom for each effect are listed in the Appendix). First, $U(m, l{:}\ M, L)$ was highly significant, $-2\log\lambda = 273.55$, $p < .005$, demonstrating a significant amount of structure in reports of heaviness and length that was contingent on the stimulus variables of mass and length. There were significant relations between each response variable and the appropriate stimulus variable: $U(m{:}\ M)$, $-2\log\lambda = 148.79, p < .005$; $U(l{:}\ L)$, $-2\log\lambda = 126.00, p < .005$. There was no overall correlation between responses, $U(m{:}\ l) = 0$, and there was no significant relation between the responses after the stimulus effects were removed: $U_{ML}(m{:}\ l)$, $-2\log\lambda = 0.91, p > .10$. No other contributors to the structure of reports of heaviness and length were significant. The partial contingent uncertainties were both nonsignificant: $U_L(l{:}\ M)$, $-2\log\lambda = 1.79, p > .10$; $U_M(m{:}\ L)$, $-2\log\lambda = 1.07$, $p > .10$. Thus, these data met Ashby and Townsend's (1986) criterion of zero partial contingent uncertainties.

The results of the separability test are shown in the next to last column of Table 6. All four pairs of conditional probabilities were statistically equal ($p > .60$). The results of the test of sampling independence are shown in Table 7. Responses of perceived heaviness and length were independent for each stimulus; none of the pairs of probabilities was significantly different ($p > .60$). The converging evidence from all of these tests supported a conclusion of perceptual independence between perceptions of weight and length.

Independence of perceived weight and width. The number of times that response $m_i w_k$ (where the subscripts refer either to greater than or less than the standard on that dimension) was made in each of the four conditions is tabulated in Table 4. The results of Garner and Morton's (1969) uncertainty analysis are shown in Table 5 (the degrees of freedom for each effect are listed in the Appendix). First, $U(m, w{:}\ M, W)$ was significant, $-2\log\lambda = 176.28, p < .005$, showing a significant amount of structure in reports of heaviness and width that was contingent on the stimulus variables of mass and width. The relations between each response and the appropriate stimulus variable were significant: $U(m{:}\ M)$, $-2\log\lambda = 121.29, p < .005$; $U(w{:}\ W)$, $-2\log\lambda = 44.58, p < .005$. There was no direct contingency between responses, $U(m{:}\ w) = 0$, and there was no significant relation between the responses after the stimulus effects were removed: $U_{MW}(m{:}\ w)$, $-2\log\lambda = 1.99, p > .10$.

One of the partial contingent uncertainties, $U_M(m{:}\ W)$, was significant in these data: $-2\log\lambda = 11.21$, $p < .005$. These data, then, failed to meet Ashby and Townsend's (1986) criterion of zero partial contingent uncertainties. Because the separability test assesses a similar characteristic in the data, it was conducted to seek converging evidence against perceptual or decisional separability. The separability test on perceived width in the last column of Table 6 provided similar results to the test of the partial contingent uncertainties. Participants were not responding equivalently across conditions and did not, therefore, appear to separate

Table 7
Results of the Test of Sampling Independence From Equation A2 on Perceived Heaviness (m) and Perceived Size (b) in Experiment 2

	$b = v$			$b = l$	$b = w$
Probability	$B = V$	$B = L$	$B = W$	$B = L$	$B = W$
M_-B_-					
$P(m_-b_-)$	0.72	0.75	0.69	0.86	0.58
$P(m_-)\cdot P(b_-)$	0.73	0.76	0.69	0.86	0.59
z	0.09	0.17	0.00	0.00	0.15
M_-B_+					
$P(m_-b_+)$	0.74	0.78	0.69	0.83	0.81
$P(m_-)\cdot P(b_+)$	0.71	0.76	0.66	0.84	0.81
z	−0.15	−0.11	−0.14	0.24	0.00
M_+B_-					
$P(m_+b_-)$	0.61	0.69	0.53	0.92	0.61
$P(m_+)\cdot P(b_-)$	0.62	0.69	0.54	0.92	0.61
z	0.11	0.00	0.13	0.00	0.00
M_+B_+					
$P(m_+b_+)$	0.68	0.63	0.75	0.94	0.75
$P(m_+)\cdot P(b_-)$	0.70	0.64	0.77	0.95	0.74
z	0.10	0.16	0.12	0.50	−0.16

Note. All $ps > 0.6$.

the effects of the width and the mass manipulations. Because converging evidence was found between the tests of separability and partial contingent uncertainties, the results of the test of sampling independence in the last column of Table 7 were logically unrelated to perceptual independence. It is important to note that the hypothesis of perceptual independence of weight and width was not rejected. It was simply the case that there was no test for perceptual independence under these conditions. It is also possible that there was a lack of separability for mass and width where both variables contribute to perceived heaviness.

Discussion

A complete identification experiment was conducted for the purpose of applying Ashby and Townsend's (1986) analysis of perceptual independence for perceived weight and size. A number of tests were performed to evaluate perceptual independence and to distinguish that from perceptual separability. These tests offered the possibility of drawing strong conclusions about perceptual independence. The data from Experiment 2 demonstrated that, in certain instances, there are independent perceptions of size and weight. In particular, the perceptions of length and volume were independent of perceived weight. This conclusion, however, could not be made in all cases. Specifically, a conclusion of perceptual independence was not possible in the case of perceived width or in the limiting case of perceived volume contingent only upon variations in the width of the stimuli.

Across variations in both length and width, perceived weight and volume were dependent only on the appropriate stimulus variables and were independent of each other. The results of the analyses of Garner and Morton (1969) and Ashby and Townsend (1986) supported this conclusion. A conclusion of perceptual independence for weight and volume was also supported in the limiting case in which the physical variation of volume resulted solely from length manipulations. In the other limiting case of purely width variations, it was not possible to make any definite conclusions about perceptual independence. This fact, however, should not detract from the conclusion of perceptual independence of weight and volume in the general case. Independent perceptions may fail to exhibit independence for every possible stimulus configuration (see Ashby & Townsend, 1986).

A conclusion of perceptual independence was also made for perceived weight and length. Each perception was only contingent upon variations in the appropriate stimulus variable. The tests of decisional and perceptual separability showed that participants had no trouble separating the effects of mass manipulations from the effects of length manipulations. Finally, these two responses were shown to be independent. The analysis of these data, then, allowed for a strong conclusion of perceptual independence.

A conclusion of perceptual independence, however, could not be made from the perceived weight and width data. Garner and Morton's (1969) uncertainty analysis showed a significant effect of physical width on perceived heaviness after the effects of mass had been partialed out. This result combined with the results of the separability test indicated that there was no test for the presence or absence of perceptual independence for these data. No conclusion of perceptual independence or dependence was possible. Although perceptual dependence remains a possibility, these results could have also been the result of a lack of perceptual separability. That is, the physical properties of mass and width may have both contributed to perceived heaviness.

Experiment 3

The patterns of independence and separability from Experiments 1 and 2 provided a rough categorization of the perceptions that are available to an observer when an object is held and lifted. There were, in the cases of perceived length and volume, unique perceptions of size and weight. The relation between perceived width and perceived weight, however, is still unclear. The possibility that these two perceptions may not be separable raises an important issue: The physical properties of width, volume, and length are all geometric properties and are therefore not directly available as sources of stimulation to a perceptual system that responds to mechanical stimulation. The implication, then, is that when these perceptions lack separability, the lack may be the result of their mutual dependence on mechanical (that is, mass-based) stimulation. When two perceptions have been shown to be independent and separable, however, there should then be independent patterns of stimulation supporting them. In Experiment 3 a magnitude estimation paradigm was used to investigate the possibility that the observed patterns of perceptual separability and independence could be related to the stimulus properties supporting these perceptions.

Recent research has demonstrated that perceptions of length, width, and weight are all functions of the structured mechanical energy distribution associated with hefting and wielding. In particular, this research has shown that these perceptions are a function of the resistances that the object presents to the rotational accelerations during lifting and holding (e.g., Amazeen & Turvey, 1996; Solomon & Turvey, 1988; Turvey, Burton, Amazeen, Butwill, & Carello, 1998). These resistances to rotational accelerations are termed rotational inertia, I. Rotational inertia ($I = \Sigma md^2$) is a function of the sum of the constituent masses, m, multiplied by the square of the distance, d, from the mass to the point of rotation. Therefore, variations in geometric properties such as length, width, and volume can all produce variations in the mechanical property I through the resulting variations in d. Perceptions of length and width are available during lifting and holding because they are each a function of I (e.g., Fitzpatrick, Carello, & Turvey, 1994; Turvey et al., 1998).

Finding patterns in I that could be related to the observed patterns of separability and independence requires more detail on I. Because rotations can occur around any of three orthogonal axes, I is quantified by the inertia tensor, I_{ij}. I_{ij} is a nine-valued matrix representing the pattern of resistances that the object presents to rotational accelerations in the various planes of motion. The subscripts *i* and *j* each refer to one of the three axes in 3-D space so that each value in the matrix I_{ij} represents the resistance to acceleration in the plane defined by the pair of subscripts. Because the three coordinate axes could be oriented in an infinite number of directions, each resulting in different components of I_{ij}, the axes for the present analysis were chosen to be the symmetry axes of the object in the hand, the eigenvectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ (the subscripts refer to each of the three axes in 3-D space). There will always be at least one set of symmetry axes for every point of rotation. Research has shown that these are the particular coordinate axes used in the perception of the orientation of limbs and hand-held objects (e.g., Pagano & Turvey, 1995). When I_{ij} is calculated for these particular axes (through diagonalization), the nine-valued matrix is reduced to three values, termed the *eigenvalues* (I_1, I_2, and I_3) of I_{ij}. The eigenvalues of I_{ij} represent the magnitudes of resistance about each of three axes (indicated by the eigenvalues subscript) that the object will present to the forces imposed by the actor-observer. In other words, the three eigenvalues reflect the rotational equivalent of mass for rotations about the three axes in 3-D space. For many objects that are longer than they are wide, increases in length produce increases primarily in I_1, whereas increases in width produce increases in I_3. More detailed discussions of rotational inertia and its calculation can be found in Goldstein (1980), Kibble (1985), and Symon (1971).

Research has shown that not only are perceived length and width functions of the eigenvalues of I_{ij}, but they are each a different function of I_{ij}. In particular, perceived length scales positively to I_1 and, in certain cases, negatively to I_3 (Carello, Fitzpatrick, Flascher, & Turvey, 1998; Fitzpatrick et al., 1994; Pagano, Fitzpatrick, & Turvey, 1993; Pagano & Turvey, 1993; Solomon, Turvey, & Burton, 1989a, 1989b; for a review, see Turvey & Carello, 1995b). Likewise, the perception of extent along an orthogonal axis (i.e., perceived width) has been shown to scale positively to I_3 and, in certain cases, negatively to I_1 (Turvey et al., 1998). Similarly, perceived heaviness is also a function of an object's inertial properties where, in addition to the effects of mass (linear inertia), the effects of size on perceived heaviness (commonly known as the size–weight illusion) appear to follow from the additional dependence of perceived heaviness on I_{ij} (Amazeen, 1997; Amazeen & Turvey, 1996). Specifically, perceived heaviness appears to scale positively to I_1 and negatively to I_3.

Within the context of an ecological analysis of these phenomena, the particular pattern in I_{ij} that appears to support each perception could be interpreted as the information for that perception (Gibson, 1966, 1979). Gibson argued that environmental properties impose structure on some form of energy (light, sound, mechanical) that can be transduced and that these patterns serve as information to the observer about the environment. Each unique perception is characterized by a unique information–perception relation. Similarly, Stevens (1960, 1962, 1970) argued that each perception is characterized by a particular psychophysical relation and that these psychophysical relations all conformed to a power function. The general form of the power function is $\Psi = k\phi^{\beta}$, where Ψ is the perception of physical property ϕ raised to the power β and scaled by constant k (see Stevens, 1970). Each kind of perception can be characterized by a unique power function (i.e., by a unique exponent). Research has shown that perceptions across all modalities conform to some power function (Stevens, 1960, 1962, 1970).

The identification of different exponents on two power laws can be taken as evidence of different perceptual processes or mechanisms (Stevens, 1960). Within the present context, the mechanism for perception is considered to be the functional mechanism of a specific information–perception relation. Given the arguments of Stevens (1960, 1962, 1970), the expectation was that each information–perception relation would conform to a power function. Therefore, the set of inertial parameters supporting each perception and the exponents defining the form of the information–perception relation were sought in Experiment 3. Each perception was reported by means of a magnitude estimation paradigm. The expectation was that independent and separable perceptions should be functions of independent sets of parameters, different exponents, or both. A lack of separability should be represented by some overlap in the information–perception relations.

Method

Participants. Thirteen undergraduate students (6 men, 7 women) at the University of Connecticut participated in this experiment as a means of fulfilling a course requirement. All participants were right-handed and reported no problems with the normal use of their hands or limbs.

Design and apparatus. The design and apparatus of the present experiment were identical to those of Experiment 1 with the following exceptions. Participants rated their perceptions of weight, volume, length, and width for each of the 18 hand-held stimuli (see Table 1). As in the previous experiments, they could wield but could not see these stimuli. Two ratings of each stimulus were made for each dimension. As indicated in Table 1, these stimuli varied in mass (309 g, 460 g, 660 g), volume (1,520 cc, 3,792 cc, 1,5096 cc), and style of volume change (varying volume through variations in length or width). These three independent variables collectively produce variations in I_{ij}, an alternate independent variable that was also evaluated.

I_{ij} for each stimulus (cylinder plus handle) was calculated for a point of rotation in the wrist that was taken to be displaced 6 cm horizontally from the top of the handle. The three eigenvalues were labeled according to their magnitudes of resistance: I_1 was the greatest and I_3 was the smallest. Because of the particular configuration of the stimuli in the present experiments, however, the eigenvalues could be roughly defined spatially (see Figure 1): For an object held vertically (with its handle parallel to the direction of gravity), I_1 was the resistance to rotations in the sagittal plane, I_2 was the resistance to rotations in the frontoparallel plane, and I_3 was the resistance to rotations along an axis running vertically through the object.

Procedure. Upon entering the experimental room, the participant was seated at the desk behind the curtain. All of the stimuli were hidden from view for the entire session, and the participant knew nothing about the objects except that they had wooden handles. Participants held and rated the objects in four sessions, one for each perceptual dimension. All sessions were conducted in the same day. The order of the sessions was randomly assigned to each participant, and a short break was allowed between sessions. The trials were spaced about 3–5 s apart, but participants were allowed to elect a short break between trials to avoid fatigue. Within a session, the order of the stimuli was randomized as well. The four perceptual reports were perceived heaviness, volume, length, and width. Perceptual reports were made by means of a magnitude estimation paradigm. Participants assigned a perceived magnitude to each experimental object relative to a standard object that was assigned a value of 100 on the particular dimension. The standard was presented on every trial and was identical in all sessions. Each trial consisted first of having the participant wield the standard and then the experimental object. There were no time limits on the trials. Participants were not allowed to view their hand or the object in their hand in all sessions.

Results and Discussion

Perceived heaviness. To evaluate the form of the scaling between I_{ij} and perceived heaviness, each I_{ij} was diagonalized, and the mean perceived heaviness for each stimulus for each participant was regressed against mass and the first and third eigenvalues (the first and second covaried) of I_{ij}, all in logarithmic coordinates. The multiple regression revealed that perceived heaviness was constrained by variations in the inertial parameters, $R^2(234) = .63$, $p < .0001$, perceived heaviness $\propto M^{1.08} I_1^{.06} I_3^{-.05}$ (M: $p < .0001$; I_1: $p > .05$; I_3: $p < .05$; see Table 8). The contribution of I_1 was not statistically significant in these data. However, previous experiments have identified a role for I_1 in the perception of weight (Amazeen & Turvey, 1996), and a separate experiment, in which the same stimuli and methodology were used, produced a similar scaling function—perceived heaviness $\propto m^{1.15} I_1^{0.1} I_3^{-0.04}$—but with all three exponents significant at the .05 level (Amazeen, 1997). Therefore, for the purposes of discussion, I_1 was retained as a parameter in this power function but with the caveat that its contribution was not statistically significant. The 95% confidence intervals for these coefficients were from 0.93 to 1.22 for M, -0.02 to 0.14 for I_1, and $-.1$ to -0.04 for I_3. The standardized coefficients were 0.79, 0.07, and $-.09$ for M, I_1, and I_3, respectively. For the individual participants, all $R^2(18)$ values were significant ($p < .0001$). Of the 13 participants, all exhibited a significant positive scaling to M ($p < .001$), 7 exhibited a significant positive scaling to I_1 (4 of the 7 were significant at $p < .05$), and 12 exhibited a negative scaling to I_3 (four significant at $p < .05$). These results are consistent with the common observation that perceived heaviness is largely a function of mass but that phenomena such as the size–weight illusion produce smaller but nonetheless significant variations in perceived heaviness.

Table 8
Power Functions Obtained in Experiment 3 Relating Each Perception to the Parameters in the Inertial Array

Perceptual report	Size variations	Power function
Heaviness	Volume	$\propto M^{1.08} I_1^{.06} I_3^{-.05}$
Volume	Volume	$\propto M^{.23} I_1^{.29} I_3^{.11}$
	Length	$\propto I_1^{.30} I_3^{.37}$
	Width	$\propto I_1^{.45} I_3^{.10}$
Length	Volume	$\propto M^{-.16} I_1^{.44}$
	Length	$\propto I_1^{.46} I_3^{-.22}$
Width	Volume	$\propto I_3^{.21}$
	Width	$\propto I_3^{.20}$

Note. All ps $< .0001$.

Perceived volume. These are the first reported data from participants who have been asked to scale volume by dynamic touch. That is, participants were allowed to hold and wield the objects but were not allowed to perceive volume either visually or by enclosing the objects in their hands. Therefore, an ANOVA of perceived volume as a function of the three stimulus variables of mass, volume, and style of volume change was performed to assess whether participants appeared to be perceiving variations in volume. There was a significant effect of volume, $F(2, 24) = 17.13$, $p < .0001$, without an accompanying interaction with the style of volume change ($p > .18$). The only other significant effect in this analysis was the strong effect of mass on perceived volume, $F(2, 24) = 17.26$, $p < .0001$. This effect of mass is consistent with the results of Experiment 1 in showing an effect of stimulus mass on perceived volume.

Because volume, like the other dimensions of size, is a geometric, not a mechanical, property, the stimulus property relevant to dynamic touch must be something other than volume itself. It was expected that participants would scale their responses to the inertial array, M and I_{ij}. A multiple regression was performed for each participant's mean perceived volume as a function of the inertial parameters of M, I_1, and I_3, all in logarithmic coordinates. As expected, the perception of volume by dynamic touch was constrained by the objects' inertial properties, $R^2(234) = .41$, $p < .0001$, where all three inertial parameters were significant positive contributors, perceived volume $\propto M^{.23} I_1^{.29} I_3^{.11}$ (M: $p < .01$; I_1: $p < .0001$; I_3: $p < .0005$; see Table 8). The 95% confidence intervals for these coefficients were from 0.06 to 0.40 for M, 0.20 to 0.39 for I_1, and 0.05 to 0.17 for I_3. The standardized coefficients were 0.18, 0.40, and 0.21 for M, I_1, and I_3, respectively. For the individual participants, all $R^2(18)$ values were significant ($p < .001$). Of the 13 participants, 5 exhibited a significant positive scaling to M ($p < .05$), 10 exhibited a significant positive scaling to I_1 ($p < .05$), and 7 exhibited a significant positive scaling to I_3 ($p < .05$). Two participants exhibited a significant negative scaling to M ($p < .05$), but no participants exhibited a significant negative scaling to either of the I_{ij} parameters ($p > .05$).

Experiments 1 and 2 showed that mass and size were not separable in the perception of volume (see Table 3) but that the perceptions of volume and weight were independent and that, across stimuli, the responses associated with these perceptions were also independent (see Table 5). It was expected, then, that the scaling function for perceived volume would include an M term but that a comparison of this function with that obtained for perceived heaviness should reveal differences so that the two responses do not necessarily covary across stimuli. Strictly speaking, perceptual independence, as Ashby and Townsend (1986) defined it, cannot be directly related to the information for a given perception because it is defined for a given stimulus (see Equation 1). As expected, M was a significant contributor to perceived volume in this analysis. Comparing the scaling functions for perceived heaviness and volume reveals some potential differences. In particular, differences can be seen in the signs of the coefficients on I_1 and I_3. The exponents on I_1 were positive in both scaling functions, although the exponent was not statistically significant in the perceived heaviness analysis. The exponents on I_3, however, were different. Perceived volume scaled positively to I_3, whereas perceived heaviness scaled negatively to I_3. These results demonstrate that, across stimuli, reports of perceived volume could both covary with mass (demonstrated in Experiment 1) and yet remain independent of reports of perceived heaviness (demonstrated in Experiment 2). Perceived volume covaried with mass because mass was one of the inertial parameters to which this perception was scaled. Reports of perceived volume and perceived heaviness were independent across stimuli because the forms of the two scalings were unique, despite the fact that they shared a set of parameters.

Experiments 1 and 2 suggested that there might be differences between the perceptions of volume accompanying purely length manipulations and purely width manipulations. The same multiple regression described earlier was performed separately for those stimuli that varied only in length and for those stimuli that varied only in width (see Table 8) to determine if any differences could be found in the scaling functions. The result was that there were few, if any, differences between the two scaling functions. For purely length manipulations, $R^2(117) = .48$, $p < .0001$, perceived volume $\propto I_1^{.30}I_3^{.37}$ (I_1: $p < .0001$; I_3: $p < .0005$). The 95% confidence intervals for the exponents were from 0.21 to 0.40 for I_1 and 0.18 to 0.56 for I_3. Standardized coefficients were 0.49 for I_1 and 0.30 for I_3. For purely width manipulations, $R^2(117) = .36$, $p < .0005$, perceived volume $\propto I_1^{.45}I_3^{.10}$ (I_1: $p < .0001$; I_3: $p < .005$). The 95% confidence intervals for the exponents were 0.27–0.63 for I_1 and 0.04–0.16 for I_3. Standardized coefficients were 0.42 for I_1 and 0.28 for I_3. Mass was not shown to be a significant contributor in either limiting case, presumably because mass covaried completely with I_1 and I_3 in these limited sets. A different set of stimuli would need to be designed in order to evaluate the contribution of mass to perceived volume in the cases of purely length or width manipulations. Nevertheless, participants appeared to be using a similar power function relating I_{ij} to perceived volume for both sets of stimuli. The differences noted in the previous experiments were not evident in the present data.

Perceived length. A multiple regression of each participant's mean perceived length for each stimulus on mass, I_1, and I_3, all in logarithmic coordinates, was performed. This regression revealed that perceived length was constrained by variations in I_1 and M, $R^2(234) = .35$, $p < .0001$. Perceived length scaled positively to I_1 and negatively to M, perceived length $\propto M^{-.16}I_1^{.44}$ (M: $p < .05$; I_1: $p < .0001$; see Table 8). The 95% confidence intervals for these coefficients were from -0.31 to -0.02 for M and 0.35 to 0.53 for I_1. The standardized coefficients were $-.14$ and 0.66 for M and I_1, respectively. For the individual participants, 12 of the 13 $R^2(18)$ values were significant ($p < .05$). Of the 13 participants, 10 exhibited a negative scaling to M (4 were significant at $p < .05$), and 13 exhibited a positive scaling to I_1 (12 were significant at $p < .05$). One participant exhibited a significant positive scaling to M ($p < .05$).

Experiments 1 and 2 showed that mass and length were separable in the perception of length (see Table 3), that the perceptions of length and heaviness were independent, and that, across stimuli, the responses associated with these perceptions were also independent (see Table 5). Therefore, a scaling function was expected that would be distinguished from that obtained for perceived heaviness and that would allow perceived length to vary independently from M. Previous experiments (Fitzpatrick et al., 1994) have shown that perceived length scales positively to I_1 and negatively to I_3. Because an increase in M will produce an increase in both I_1 and I_3, this scaling would allow two objects of equivalent lengths and different masses to be perceived as having similar lengths despite the differences in I_1 (Fitzpatrick et al., 1994). In other words, this scaling could support a separability of length and mass. The present experiment failed to show such a negative scaling to I_3 but, rather, showed a negative scaling to M. Because an increase in M will also produce an increase in I_1, the scaling function obtained in the present experiment could also support a separability of length and mass. Comparing the power functions for perceived length and perceived heaviness revealed a couple of differences. First, each perception was a function of different sets of parameters. Second, perceived heaviness scaled positively to M, whereas perceived length scaled negatively. These differences could support an independence of perceptual reports across stimuli.

It was hypothesized on the basis of the results from Experiments 1 and 2 that the perception of length may be a distinct kind of perception from perceived volume. These suggestions were based on differences in the perceptions of volume and length accompanying variations in stimulus length. Therefore, a multiple regression was performed on the set of stimuli that varied in length and mass only (see Table 8). Once again, perceived length was shown to be a function of I_{ij}, $R^2(117) = .48$, $p < .0001$. The exponent on I_1 was positive and significant ($p < .0001$), whereas the exponent on I_3 was negative and significant ($p < .05$), perceived length $\propto I_1^{.46}I_3^{-.22}$. This negative dependence on I_3 is consistent with previous findings (Fitzpatrick et al., 1994), perhaps because now, as in those experiments, the variations in I_3 are primarily the result of variations in M rather than variations in width. The 95% confidence intervals for the exponents were from 0.37 to 0.55 for I_1 and -0.40 to -0.04 for I_3. Standardized coefficients were 0.77 for I_1 and -0.19 for I_3.

Despite the fact that length and volume completely covaried for these stimuli, the act of making one response or another produced different scaling relations between each perception and the inertial array. For perceived volume, the exponents on I_1 and I_3 were both positive and significant. For perceived length, however, the exponent on I_1 was positive, and the exponent on I_3 was negative. This difference would result in significant differences between each report, even when the stimuli are identical.

Perceived width. The multiple regression of each participant's mean perceived width on mass, I_1, and I_3, all in logarithmic coordinates, produced a scaling relation in which only I_3 was a significant contributor, perceived width $\propto I_3^{.21}$, $r^2(234) = .23$, $p < .0001$ (see Table 8). The 95% confidence intervals for the exponent were from 0.16 to 0.27. This same result also held when the regression was performed on only those objects for which variation in volume is brought about through variations in width, perceived width $\propto I_3^{.20}$, $r^2(117) = .39$, $p < .0001$ (see Table 8). The 95% confidence intervals for the exponent were from 0.15 to 0.26. For the individual participants, 11 of the 13 $R^2(18)$ values were significant ($p < .05$). Of the 13 participants, 11 exhibited a positive scaling to I_3 (9 were significant at $p < .05$).

These results provided further insight into the relation between reports of perceived heaviness and perceived width but still did not allow for a definitive conclusion regarding independence. The observed scaling functions for perceived width and heaviness were, in fact, different, but it is not clear whether sufficient differences existed to make the reports independent. Relatedly, these scaling functions could signal perceptual independence if they supported a lack of perceptual separability (i.e., if there were covariations in the information for each perception across stimuli leading to a between-stimulus response correlation). Independence could not be established in Experiment 2 because of a lack of separability, but the analysis only requires decisional, not perceptual, separability to hold (see Ashby & Townsend, 1986, for a proof). Therefore, the fact that variations in the only stimulus variable related to perceived width, I_3, produce covariation in perceived width and heaviness may be related to perceptual independence. Definitive conclusions of perceptual independence in this case require further empirical investigation.

General Discussion

Because of the existence of a size–weight illusion, much research on weight perception has been based on the assumption that perceived weight is not independent of perceived size. Recent research, however, has revealed that something other than perceptual dependence may be responsible for the size–weight illusion, namely, the dependence of perceived heaviness on I_{ij} (Amazeen, 1997; Amazeen & Turvey, 1996). Variations in size (volume, length, or width) can affect perceived heaviness because I_{ij} is a function of size. To the extent that observed perceptual phenomena (such as the size–weight illusion) can be shown to be a function of a particular stimulus parameter, then these phenomena are not necessarily the result of a perceptual coupling. The suggestion that perceived weight and size may be independent motivated the present series of experiments.

Perceptual Independence

For the present experiments, Ashby and Townsend's (1986) definition of perceptual independence was adopted. Perceptual independence was defined as a statistical independence between two perceptions for a particular stimulus (see Equation 1). This was distinguished from the related concept of perceptual separability, which refers to the effect, across stimuli, of one physical variable (such as volume) on a noncorresponding perception (such as weight). Both of these effects could produce a lack of independence in perceptual reports, but their interpretation would be very different. A lack of perceptual independence would imply some sort of percept–percept coupling, whereas a lack of perceptual separability would imply that the stimulus parameters, or information, supporting these perceptions are not independent.

The issue of separability was raised in Experiment 1 by investigating the relations between reports of perceived heaviness and size and the physical dimensions of mass and size. Reports of heaviness, length, and width were contingent upon only the appropriate stimulus variable. These perceptions appeared to be separable. Perceived volume, however, was shown to be strongly contingent upon variations in mass. Perceptual independence was tested in Experiment 2 by means of the formal tests developed by Ashby and Townsend (1986). These data demonstrated that there are, in certain instances, independent perceptions of size and weight. In particular, the perceptions of length and volume were independent of perceived weight. However, a conclusion of perceptual independence was not possible in the case of perceived width or in the limiting case of perceived volume contingent upon only variations in the width of the stimuli. In these instances, a lack of decisional or perceptual separability disallowed a test of perceptual independence for these data; perceptual independence could neither be concluded nor rejected. These results, along with Reed's (1994) demonstration of dependent perceptions of shape and texture, have begun to identify the various independent categories of perceptions that are available to the haptic perceptual system (tactual perception).

In Experiment 3 a magnitude estimation paradigm was used to investigate the possibility that the observed patterns of separability and independence could be related to the stimulus properties supporting these perceptions. In particular, patterns in the inertial array (constituted by mass and the eigenvalues of I_{ij}) were sought that could support these perceptions. These stimulus patterns can be considered information, in the ecological sense, because they are patterns in an energy array that are functions of certain environmental properties. It was expected that independent and separable perceptions would be supported by independent information (i.e., by unique sets of parameters, unique exponents scaling magnitude estimates of perception to those parameters, or both). In the case of perceived heaviness and perceived volume, the two perceptions were both supported by the same set of parameters, but the exponents on these parameters were not identical. These two facts could explain how, across stimuli, reports of perceived volume can both covary with mass but remain independent of reports of perceived heaviness. The power function for perceived length, however, appears to support the observed separability as well as the independence of perceptual reports. In the case of perceived width, the scaling function was, in fact, different from that obtained for perceived heaviness, but it was not clear whether it was sufficiently different to support separability and response independence. The power functions observed in the present experiment were similar to those found in previous experiments (e.g., Amazeen & Turvey, 1996; Fitzpatrick et al., 1994; Turvey et al., 1998), but future research should examine the possible effects of the unequal ranges of stimulus dimensions (mass, volume, length, width, I_1, and I_3) in the present experiments.

Perception Categories as Action Categories

A primary goal of this research has been to investigate the existence of perceptual separability and independence for perceived size and weight and to relate those findings to the information for dynamic touch. It has been assumed in these discussions that the analysis of perceptual independence would reveal independent perceptions of the corresponding physical properties. It is notable, however, that these perceptions were systematically different from the physical property that the individuals reported (mass, volume, length, and width). The most relevant findings in this respect were the differences between reports of volume and length or width when the stimuli covaried on those dimensions. Nevertheless, these perceptions appeared to be supported by particular patterns in the inertial array. Although these patterns may be the information for dynamic touch, the present results raise questions regarding specificity and the content of the information that are central to any definite conclusions regarding information. Similar results have also been found in the study of perceived length and grip position by dynamic touch (Pagano, Carello, & Turvey, 1996). Pagano et al. (1996) noted that although these perceptions often corresponded to the actual geometric properties, the perceptions did not always necessarily do so. Further, when they did not correspond, it was not simply error variance; the lack of correspondence was regular and consistent. In these instances, when a perception appears to correspond to information but not to the expected environmental property, another type of labeling may be most appropriate.

Pagano et al. (1996) offered an alternate hypothesis in their analysis of perceived length and grip position. They suggested that the perceptions could be related to those properties that are important in controlling hand-held objects. This same hypothesis may be relevant to the present data as well. Instead of indexing purely perceptual categories, perhaps two independent perceptions could be said to index two (possibly independent) categories or kinds of action. The information in the inertial array may be the specification of action-relevant properties rather than geometric properties. A perceptual report, then, could be an index of the perception that the observer-actor has of the possible or required actions in a particular context. Very similar conclusions have previously been offered for weight perception (Bingham, Schmidt, & Rosenblum, 1989), and empirical support for this type of conclusion for perceived reachability has already been provided (Heft, 1993). This type of argument—that a perception may be of an action rather than a physical property—has also been offered with respect to the neurophysiology of the cortical visual system (Goodale, 1994; Goodale & Milner, 1992; Milner & Goodale, 1993). These authors pointed out that lesions to certain cortical areas can produce selective deficits either in object identification or in movement control relative to that object. This leads to the conclusion that there may be somewhat independent visual pathways governing perceptions of "what" and "how." Although no such definite conclusions could be made from the present data, future research should investigate the possibility that these perceptions are more about controlling an object than about identifying the object's physical characteristics.

The inertial model of dynamic touch would be well suited for such an analysis. The perceptions have been shown to correspond to patterns in the inertial array. By definition, this inertial array is the pattern of resistances that an object will make to the forces imposed by an observer; it is the coupling between forces and motions. Information in the inertial array, then, could specify to the observer the requisite patterns of forces needed to produce a particular motion. That is, the perceptual reports in these experiments could be indexing perceptions of action-relevant properties rather than perceptions of geometric properties. Bingham et al. (1989) suggested that the perception of weight may actually be the perception of "throwability." Similar hypotheses regarding the categories defined in the present series of experiments could be fruitful in future investigations.

Conclusions

There are, in certain cases, independent perceptions of size and weight by dynamic touch. Specifically, perceptions of volume and length were independent of perceived weight. In these instances, independent and separable perceptions were functions of independent information (defined as unique sets of parameters, unique exponents scaling magnitude estimates of perception to those parameters, or both). Although not necessarily indicative of perceptual dependence, there was a lack of response independence for perceptions of heaviness and width and for perceptions of heaviness and volume when volume was varied only through variations in width. It may be that this particular response correlation, rather than a general dependence of perceived weight on volume, is what has been referred to as the size–weight illusion. The exact cause, however, is still uncertain. It may be the result of a lack of perceptual independence or separability. Throughout the present series of experiments, there were cases in which the perceptions appeared to deviate from the actual physical property but to still be a function of the available information. These phenomena raised an important issue, namely, if the information does not necessarily specify a physical property, then what does it specify? The suggestion was offered that these perceptions may be indexing possible or required actions with regard to the object. Thus, although the present data allowed for certain conclusions regarding perception by dynamic touch, they also highlight the need to further understand the roles of activity and of the size–weight illusion in weight perception in particular and in tactual perception in general.

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Appendix

Uncertainty Analysis

Analysis of Experiment 1: Garner (1962)

The data in Experiment 1 were matrices indicating the number of times, n_i, that a particular category i of response type y was made. Such frequency counts can easily be converted into probability matrices of the same form in which each entry is the probability of response y_i, $p(y_i) = n_i/n$. Analysis of these data requires the use of nonmetric statistical techniques, but the within-groups design makes these data inappropriate for the common chi-square analysis of independence. A number of statistical measures for these types of data (categorical responses to orthogonal categorical predictors) have been developed (Garner, 1962; Garner & McGill, 1956; Garner & Morton, 1969; McGill, 1954; Quastler, 1955) by means of Shannon's measure of information (1948; Shannon & Weaver, 1949). The focus of these analyses was to measure the structure in the observed responses that is provided by one or more of the predictors. The method of partitioning the structure in y into meaningful components was first developed by McGill (1954). Presentation of this analysis, however, also follows the further elaborations of Garner (1962; Garner & McGill, 1956; Garner & Morton, 1969).

Preliminaries. Shannon's information metric quantifies structure, or information, as an amount of reduced uncertainty. The specific measure, termed *uncertainty* ($U(y)$), is given by the following:

$$U(y) = -\sum_{i=1}^{k} p(y_i)\log_2 p(y_i), \tag{A1}$$

where response y has k alternatives. The uncertainty measure is a form of nonmetric variance (Garner, 1962). Thus, the total amount of uncertainty, $U(y)$, is maximum for a matrix with equal probabilities of responding for all y_i. Uncertainty is reduced as a pattern or structure emerges in the responding. The amount of uncertainty reduction, then, is a measure of observed structure—this is Shannon's hypothesis.

Equation A1 defines the uncertainty for a univariate matrix. The data in the present experiment, however, were multivariate matrices indicating $p(y_i)$ as a function of one or more predictor variables. Equation A2, then, is the multivariate equivalent of Equation A1:

$$U(y, X) = -\sum_{y,X} p(y, X)\log_2 p(y, X). \tag{A2}$$

Equation A2 returns the joint uncertainty, $U(y, X)$, for a bivariate matrix defined over response variable y and predictor variable X. $U(y, X)$ is the total observed uncertainty. Lowercase letters denote response variables, and uppercase letters denote predictor variables. It should be noted that Equation A2 holds for any set of variables (response or predictor), but y and X were so denoted here to be consistent with the form of the present results.

Measuring total structure. Obtaining a measure of structure requires calculating the maximum possible uncertainty for the data matrix and subtracting from it the observed uncertainty (i.e., measuring uncertainty reduction). The maximum possible uncertainty is termed *maximum joint uncertainty,* $U_{max}(y, X)$. $U_{max}(y, X)$ is the uncertainty constrained by the marginal probabilities of the matrix and is calculated by applying Equation A1 to the marginal probabilities themselves. An equivalent result is obtained by generating the expected matrix of probabilities as one would for a chi-square analysis and then applying Equation A2 to that matrix.

Up to this point, the terms have been generated under the assumption of a bivariate matrix. In the present experiment, however, the response y was a function of two independent variables. Therefore, where X was used before, the set of predictor variables, A and B, is substituted. The total constraint, $U(y: A, B)$, is thus obtained by subtracting the total joint uncertainty, $U(y, A, B)$, from the maximum uncertainty, $U_{\max}(y, A, B)$.[A1] The multiple contingent uncertainty, $U(y: A, B)$, represents the relation between the response variable and the set of stimulus variables.

Partitioning uncertainty. The total constraint, $U(y: A, B)$, can be partitioned to determine not only the total amount of structure but the source(s) of that structure as well. Following Garner (1962), $U_{\max}(y, A, B)$ and $U(y, A, B)$ can be expanded and rearranged to give

$$U(y) = U(y: A, B) + U_{AB}(y), \tag{A3}$$

where the total uncertainty in y, $U(y)$, can be partitioned into that attributable to the two predictor variables, $U(y: A, B)$, and the remaining error, $U_{AB}(y)$. The first term on the right is the multiple-contingent uncertainty (structure contingent upon a set of predictor variables), and the second term is the error uncertainty. The subscripted variables in $U_{AB}(y)$ indicate that the uncertainty is what is left after the effects of A and B have been removed.

The multiple-contingent uncertainty, $U(y: A, B)$, is the term of primary significance because it represents the effects of the independent variables on the dependent variable. In Experiment 1, $U(y: A, B)$ was partitioned in a way similar to that used in an ANOVA. This partitioning, shown in Equation A4, divides $U(y: A, B)$ into the uncertainty attributable to each predictor alone plus the interaction uncertainty, $U(\overline{yAB})$,

$$U(y: A, B) = U(y: A) + U(y: B) + U(\overline{yAB}). \tag{A4}$$

Interpretation of these terms is the same as that of the equivalent terms in an ANOVA. In fact, uncertainty analysis can be shown to be an analogous technique, with the added benefit that no assumptions are made about the underlying metric (Garner & McGill, 1956). For this reason, it has been argued that an uncertainty analysis is appropriate not only for nonmetric data but also for metric data when assumptions are not (or cannot be) made about the underlying metric.

With the appropriate partitioning and calculation of terms, the significance of each contingent uncertainty can be evaluated by means of the likelihood ratio, λ (Garner & McGill, 1956; McGill, 1954; Miller, 1955; Miller & Madow, 1954). Recall that each contingent uncertainty represents the amount of relation between the response variable and one or more predictor variables. The alternate hypothesis, then, is defined using the appropriate form of Equation 1—that is, the response is independent of the predictor(s). A ratio between these two hypotheses is represented by λ. A test of this ratio is based on the fact that $-2\log\lambda$ is distributed as a chi-square when the variables are independent (Fienberg, 1980; Mood, Graybill, & Boes, 1974; Plackett, 1974), and the variables are independent when the relevant contingent uncertainty, $U(y: X)$, equals zero. Furthermore, it can be shown that $1.3863nU(y: X) = -2\log\lambda$ (Garner & McGill, 1956; McGill, 1954; Miller, 1955; Miller & Madow, 1954). Thus, the test of $U(y: X) > 0$ involves calculating $-2\log\lambda$ and evaluating it against the chi-square distribution. In Experiment 1, uncertainties were evaluated against the chi-square distribution with 3 degrees of freedom (*df*) for $U(y: A, B)$ and 1 *df* for each of the other sources (see McGill, 1954). Because of the existence of small or zero observed frequencies in some cells of the matrices, the improved likelihood ratio test was used to make $-2\log\lambda$ a better approximation of chi-square (Fienberg, 1980; Williams, 1976). This improved test involves multiplying $-2\log\lambda$ by q_{min}^{-1} where $q_{min} = 1 + (y + 1)(X + 1)/6n$. It is noteworthy that this test cannot be conducted for interaction uncertainties because they are distributed as the difference between chi-square distributions (McGill, 1954).

Analysis of Experiment 2: Garner and Morton (1969)

Garner and Morton (1969) extended the uncertainty analysis of McGill (1954) and Garner (1962) to the case of two dependent variables for the specific purpose of evaluating perceptual independence. The essentials of this analysis are identical to those presented above. Therefore only the necessary elaborations are presented here. In Experiment 2, the contingencies between perceived heaviness, m, perceived size, b, and the physical variables of mass, M, and size, B, were measured and evaluated statistically. As before, b (and B) can stand for perceptions of (and physical variations in) either volume, length, or width.

The multiple-contingent uncertainty, $U(m, b: M, B)$, representing the total effects of the set of stimulus variables on the set of response variables, can be expressed as

$$U(m, b: M, B) = U(m: b: M: B) - U(m: b) - U(M: B). \tag{A5}$$

Here $U(m, b: M, B)$ is equal to the total constraint among all four variables, $U(m: b: M: B)$, minus the relation between the two perceptual variables, $U(m: b)$, and the relation between the two stimulus variables, $U(M: B)$. This last term is only included for completeness and generality; in all standard treatments, the stimulus variables are orthogonal and this relation is, therefore, zero. Following Garner and Morton (1969), the total constraint, $U(m: b: M: B)$, can be partitioned as follows to reflect the relevant relations between the variables in the set:

$$U(m: b: M: B) = U(m: M) + U(b: B) + U_B(b: M) + U_M(m: B) + U_{MB}(m: b). \tag{A6}$$

The first two terms on the right are the direct contingencies measuring the amount of relation between each response variable and the appropriate stimulus variable. The next two terms are the partial contingent uncertainties. These terms reflect the amount of relation between a response variable and the inappropriate stimulus variable after the effects of the appropriate stimulus variable have been partialed out.

The final term on the right, $U_{MB}(m: b)$, is the partial contingency between response variables and is related to the direct contingency between response variables, $U(m: b)$. The direct contingency, $U(m: b)$, is calculated by means of the matrix of marginal probabilities for the response variables (i.e., the entire matrix is collapsed across stimulus variables). The partial contingency between response variables, however, is obtained by calculating $U(m: b)$ at each level of M_qB_r and then taking a weighted mean (see Garner, 1962). Thus, $U(m: b)$ represents an overall relation between response variables, and $U_{MB}(m: b)$ differs from $U(m: b)$ when that particular relation varies across levels of the stimuli. Garner and Morton (1969) argued that $U_{MB}(m: b)$ is simply error variance and that $U(m: b)$ is

[A1] When two or more variables are being treated as one, a comma is used between them. The comma should be read simply as *and*. Colons, however, indicate a relation between two or more variables and should be read as *a function of* or *contingent upon*.

the term most directly related to perceptual independence. However, if there is a relation between m and b and an interaction between terms such that this relation varies across levels of the stimulus, then only $U_{MB}(m\colon b)$ reflects this relation. This is because the varying relations may completely or partially cancel each other out when collapsed across stimulus levels, resulting in a lowered $U(m\colon b)$. Both terms, then, should be evaluated together to determine the nature of the relation between response variables.

The complete partitioning of terms relevant to perceptual independence, then, is achieved by substituting Equation A6 for Equation A5. The degrees of freedom for these terms in Experiment 2 are as follows (see McGill, 1954): 1 *df* each for $U(m\colon M)$ and $U(b\colon B)$; 2 *df* each for $U_B(b\colon M)$ and $U_M(m\colon B)$; 4 *df* for $U_{MB}(m\colon b)$; 11 *df* for $U(m\colon b\colon M\colon B)$; 1 *df* for $U(m\colon b)$; and 9 *df* for $U(m, b\colon M, B)$. As a set, these uncertainties specify the patterns of dependence of the perceptual variables on each other and on the set of stimulus variables.

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